Analytical Delay Models for Signalized Intersections

Ali Payidar Akgungor and A. Graham R. Bullen

INTRODUCTION

Delay is the most important measure of effectiveness (MOE) at a signalized intersection because it relates to the amount of lost travel time, fuel consumption, and the frustration and discomfort of drivers. Delay also can compare the performances of an intersection under different control, demand and operating conditions. The accurate prediction of delay is, therefore, very important, but its accurate estimation is difficult due to random traffic flows and other uncontrollable factors.

Delay can be estimated by measurement in the field, simulation, and analytical models. Of these methods, analytical estimation is the most practical and convenient. In estimating of delay at signalized intersections, a number of analytical models have been proposed and developed using different assumptions for various traffic conditions.

Many stochastic steady-state delay models use the assumptions that arrivals are random and departure headways are uniform, but these assumptions are generally unrealistic. Stochastic steady-state delay models are applicable only for under-saturated conditions and they predict infinite delay when arrival flows approach capacity. Deterministic models are more realistic for predicting delay for over-saturated conditions, but these models ignore the effect of randomness in traffic flow.

Time dependent delay models have been developed to overcome the deficiencies in both stochastic steady state and deterministic delay models. These models combine the stochastic steady state and deterministic models using the co-ordinate transformation technique. They provide more realistic delay models.

There are three different time dependent delay models (Australian, Canadian and the Highway Capacity Manual (H.C.M.)) commonly used to estimate delay at signalized intersections. There is a delay parameter k in all of these models that is fixed but this k parameter and does not account for the effects of variable traffic demands and variable time periods of analysis.

This paper develops time dependent delay models for the estimation of delay at signalized intersections for variable demand and time conditions. The delay parameter k in these models is a function of degree of saturation and analysis time period.

BACKGROUND

Average total delay experienced by vehicles at an intersection controlled by a pre-timed traffic signal consists of uniform, random overflow and continuous overflow delays. Many analytical models with varying assumptions have been developed to estimate this traffic signal delay. Even some of the time dependent delay models, however, have not been able to estimate delay accurately for over-saturated conditions. This includes the H.C.M. delay model, which is popular and widely used.

The H.C.M. delay model yields reasonable results for under-saturated conditions but compared to other delay models, predicts higher delays for over-saturated conditions. The difference between the H.C.M. delay model and other delay models increases with increasing degree of saturation. Therefore, delay estimates for higher values are not recommended. [1, 2]* The H.C.M. delay model was derived for a time period of 15 minutes and hence, the estimation of delay using this model is limited to time periods of 15 minutes duration.

The level of delay at a signalized intersection is a function of many parameters including the capacity, the traffic volume, the amount of green time available, the degree of saturation, the analysis time period, and the arrival patterns of vehicles.

Time dependent delay models include a delay parameter in their overflow delay component known as the *k* variable, which describes the arrival and service conditions at the intersection. Degree of saturation (*x*), which is the ratio of arrival flow to capacity, and analysis time period (*T*) affect directly the delay. Therefore, the delay parameter *k* can be expressed as a function of degree of saturation and analysis time period.

A Historical Perspective of Delay Models

Over the past 40 years, many models have been developed to estimate vehicle delay at signalized intersections. One of the first delay was Wardrop's [3] delay expression developed in 1952. Wardrop assumed that vehicles enter the intersection with uniform arrivals. In this model, Wardrop reported that the term 1/2s is generally small compared with r and can be neglected. The Wardrop's expression is expressed as:

$$d = \frac{(r - \frac{1}{2s})^2}{2C(1-y)} \qquad (1)$$

where

d = average delay per vehicle in sec,
r = the effective red time in sec,
s = saturation flow on the approach in vps or vph,
C = cycle length in sec,
y = flow ratio.

Three more representative models estimate delay at signalized intersections have been proposed by Webster [(4)], Miller [(5)] and Newell [(6)], while Hutchinson, [(7)] Sosin [(8)] and Cronje [(9)] have numerically compared these delay expressions.

The model developed by Webster [(4)] in 1958 is the basic delay model for signalized intersections. Webster assumes that arrivals are random and departure headways are uniform, and his expression is as follows:

$$d = \frac{C(1-\lambda)^2}{2(1-\lambda x)} + \frac{x^2}{2q(1-x)} - 0.65\left(\frac{C}{q^2}\right)^{1/3} x^{(2+5x)} \qquad (2)$$

where

λ = green ratio,

x = degree of saturation,

q = flow rate in vph.

The first two terms in the Webster's expression are theoretical while the last term is an empirical correction factor. The first term in this expression is delay due to a uniform rate of vehicle arrivals and departures. The second term is the random delay term, which accounts for the effect of random arrivals. Webster found that the correction term, which is the last term in the expression, represents between 5 and 15 percent of the total delay. For practical usage, the correction term often is eliminated and replaced by a coefficient of 0.9 applied to the first and second delay terms. Webster's simplified expression is:

$$d = \frac{9}{10}\left(\frac{C(1-\lambda)^2}{2(1-\lambda x)} + \frac{x^2}{2q(1-x)}\right) \qquad (3)$$

One of the major issues in developing delay models at signalized intersections is the estimation of overflow delay. The difficulty of obtaining simple and easily computable formulae for overflow delay has forced to analysts to search for approximations and boundary values. An obvious lower boundary value of overflow delay is zero, which applies to low traffic intensities. Miller suggested that the magnitude of the overflow delay is insignificant when the degree of saturation is less than 0.5. [(10)]

For an upper boundary value, Miller [(5)] found an approximation given by:

$$\frac{\exp\left[\frac{(-1.33)\sqrt{sg}\,(1-x)}{x}\right]}{2(1-x)} \qquad (4)$$

where

g = effective green time in sec.

Miller developed, in terms of the overflow, two expressions assuming that the queue on the approach was in statistical equilibrium and the number of arrivals in successive red and green times were independently distributed.

Miller's first delay expression, which incorporates the I ratio, is represented as follows:

$$d = \frac{(1-\lambda)}{2(1-\lambda x)}\left(C(1-\lambda) + \frac{(2x-1)I}{q(1-x)} + \frac{I+\lambda x - 1}{s} \right) \tag{5}$$

where:

I = variance to mean ratio of flow per cycle.

The first term in the expression gives the average uniform delay resulting from the interruption of traffic flow by traffic signals. The second term of the expression shows the measurement of average delay when there are vehicles left in the queue at the end of green phase. The third term causes delay to decrease when $I < 1$ or increase when $I > 1$. Note that this expression is only valid when $x > 0.5$. When x is less than 0.5, the middle term in the bracket vanishes. Miller also assumes that zero overflow is implied when the number of departures in a cycle is less than sg.

Miller found that his and Webster's expression gave similar results when I was almost equal to 1, but his model gave better agreement with measured delay in the field when I was greater than 1 [3]. Miller's second delay expression is given by:

$$d = \frac{(1-\lambda)}{2(1-\lambda x)}\left(C(1-\lambda) + \frac{\exp\left[\frac{(-1.33)\sqrt{sg}\,(1-x)}{x}\right]}{q(1-x)} \right) \tag{6}$$

Newell studied general arrival and departure distributions for delay models at signalized intersections. He expressed that the average delay experienced by vehicles as:

$$d = \frac{C(1-\lambda)^2}{2(1-y)} + \frac{IH(\mu)x}{2q(1-x)} \tag{7}$$

Where I is the variance to mean ratio of arrivals and H (μ) is a function given by the following equation:

$$\mu = \frac{sg - qc}{(Isg)^{0.5}} \tag{8}$$

The function of H (μ) was obtained by numerical integration that ranges between 1 at μ = 0 to 0.25 at μ = 1.

Cronje [9] proposed an alternative approximation for H (μ), which is expressed as follows:

$$H(\mu) = \exp\left[-\mu - (\frac{\mu^2}{2}) \right] \tag{9}$$

Where:

$$\mu = (1 - x)(s.g)^{0.5} \tag{10}$$

By comparing the results with Webster's expression, Newell introduced another supplementary correction term to improve the results for medium traffic intensities, and his expression took final form as:

$$d = \frac{C(1-\lambda)^2}{2(1-y)} + \frac{IH(\mu)x}{2q(1-x)} + \frac{(1-\lambda)I}{2s(1-\lambda x)^2} \tag{11}$$

Hutchinson [7] modified Webster's simplified expression by introducing the variable *I*. Hence, Webster's simplified expression presented in Equation (2-12) is a special case of Equation (2-3) when *I* equals 1. The expression modified by Hutchinson is as follows:

$$d = \frac{9}{10}\left(\frac{C(1-\lambda)^2}{2(1-\lambda x)} + \frac{Ix^2}{2q(1-x)} \right) \tag{12}$$

Hutchinson's analysis for these models showed that Webster's simplified expression underestimates delay when *I* is greater than 1 and the degree of saturation is high. He also pointed out that Webster's expression modified to include the *I* variable is a good alternative model to estimate stochastic delay because of its algebraic simplicity.

Van As [11] performed a study using a macroscopic simulation techniques based on the principles on Markov chains to evaluate Miller's, Newell's and Hutchinson's modifications of Webster's delay expressions. The results showed that Miller's and Newell's models do not significantly improve the estimation of delay by reason of their complexity. On the other hand, Hutchinson's modification of Webster's delay expression performed well and provided a significant improvement in estimating delay.

Van As also developed a semi-empirical formula to transform the variance to mean ratio of arrivals I_a into the variance to mean ratio of departures I_d, which is applied to Hutchinson's modification of Webster's delay expression. This semi-empirical formula is expressed as follows:

$$I_d = I_a \exp(-1.3F^{0.627}) \qquad (13)$$

where

I_d = variance to mean ratio of departures,

I_a = variance to mean ratio of arrivals.

with the factor F given by:

$$F = \frac{Q_o}{(I_a qC)^{0.5}} \qquad (14)$$

where

Q_o = average overflow queue in vehicles.

Tarko et al.[12] investigated overflow delay at a signalized intersection approach influenced by an upstream signal. In this study two overflow delay model forms, with variance to mean ratio of upstream departures and capacity differential between intersections, were evaluated using a cycle by cycle simulation model developed by Rouphail. [13]

This study showed that random overflow delay approaches zero when the upstream capacity is less or equal to the capacity at the downstream intersection. They also found that the variance to mean ratio I comes close to zero when the upstream approach is close to saturation. Therefore, Tarko et al. concluded that the upstream signal impact is not appropriately represented by the variance to mean ratio I.

For this reason, the variance to mean ratio I was dropped from the steady state model and they proposed an overflow delay model in terms of f, which is a function of the capacity differential between the upstream and downstream intersections. In the proposed overflow delay model, f is also a function of a delay parameter k. The details of these parameters are presented later in chapter 4 in a section describing the Tarko-Rouphail k variable.

Some studies have been performed based on the statistical distributions. Brillen and Wu [14] developed a new approach using Markov chain to estimate delay at signalized intersections under Poisson and non-Poisson conditions. Cronje [15] also considered traffic flow at signalized intersection as a Markov process and derived delay models for undersaturated and oversaturated conditions.

Heidemann [16] and Olszewski [17] used probability distribution functions to estimate delay at signalized intersections. In both models, the probability distributions of delay were obtained from the probabilities of queue lengths.

TIME DEPENDENT DELAY MODELS

Analytical models for the estimation of delay at signalized intersections have three delay components, uniform delay, random overflow delay and continuous overflow delay.

Uniform Delay

For uniform delay randomness in the arrivals is ignored as a constant arrival rate is assumed. The discharge rate varies from zero to saturation flow according to the following conditions: [18]

- Zero during the red interval,
- The saturation flow rate during the part of green when there is a queue,
- The arrival rate during the part of green when there is no queue.

For a degree of saturation less than 1.0 the expression for uniform delay is given by the first term in Webster's equation (2).

For over-saturated conditions the uniform delay is given by:

$$d_u = 0.5(C - g) \tag{15}$$

Random Overflow Delay

Actual vehicle arrivals vary in a random manner[18] and this randomness causes overflows in some signal cycles. If this persists for a long time period then the over-saturated conditions lead to continuous overflow delay. Akcelik[19] expressed the overflow delay component as a function of average overflow queue. The effect of the overflow depends on the degree of saturation over a given time period.

Continuous Overflow Delay

Continuous overflow delay is the delay experienced by vehicles which are unable to discharge within the signal cycle because the arrival flow is greater than capacity. Continuous overflow delay is directly proportional to the time period for analysis *T* and the degree of saturation. Continuous overflow delay is also called "deterministic overflow delay" or "deterministic delay" due to its deterministic queuing concept. The deterministic model assumes a constant arrival rate and capacity, which is determined by the fixed time operation of a signal. The model presumes that the queue length at the beginning of the analysis period is zero and increases linearly to until the end of the analysis time period.

The deterministic or continuous overflow model is a key predictor for estimation of the delay and the queue under highly congested conditions, but it is not an appropriate model for lightly congested conditions [20-21].

Time Dependent Delay Models

Time dependent delay models fill more realistic results in estimating delay at signalized intersections. They are derived as a mix of the steady state and the deterministic models by using the coordinate transformation technique described by Kimber and Hollis.[22-23] The technique was originally developed by P.D. Whiting to derive the random delay expression for TRANSYT computer program. [24]

The coordinate transformation is applied to the steady state curve, and smoothes it into a deterministic line by making the steady state curve asymptotic to the deterministic line. [22-24] Thus, time dependent delay models predict delay for both undersaturated and oversaturated conditions without having any discontinuity at the degree of saturation 1.0.

Australian Delay Model

The Australian delay model, which was derived by Akcelik,[19,25] is an approximation to Miller's delay model. The Australian delay model predicts zero overflow delay for low degrees of saturation before the overflow delay term is applied. The value of the minimum degree of saturation depends on capacity per cycle and is given a symbol x_o. The Australian delay model is expressed as follows:

$$d = \frac{C(1-\lambda)^2}{2(1-\lambda x)} + 900T\left[(x-1) + \sqrt{(x-1)^2 + 12\left(\frac{x-x_0}{cT}\right)}\right] \tag{16}$$

and

$$x_0 = 0.67 + \frac{sg}{600} \tag{17}$$

where

d = average overall delay,
C = cycle time (sec),
λ = green ratio,
x = degree of saturation,
c = capacity (vph),
x_0 = degree of saturation below which the second term delay is zero,
sg = capacity per cycle (vehicle/cycle).

Canadian Delay Model

The Canadian delay model derived by Whiting is commonly used to predict delay at signalized intersections.[26] The model , like other time dependent delay models, consists of two terms that have uniform and overflow delay terms. The Australian and Canadian delay models have a similar formulation, but different coefficients in the overflow delay term. The original form of the Canadian delay model is stated as follows:

$$d = \frac{C(1-\lambda)^2}{2(1-\lambda x)} + \frac{15t_e}{c}\left[(v-c)+\sqrt{(v-c)^2+\frac{240v}{t_e}}\right] \quad (18)$$

where

t_e = evaluation time (minutes),
v = arrival flow rate.

After some manipulation, the Canadian Delay model is expressed as:

$$d = \frac{C(1-\lambda)^2}{2(1-\lambda x)} + 900T\left[(x-1)+\sqrt{(x-1)^2+\frac{4x}{cT}}\right] \quad (19)$$

1985 H.C.M. Delay Model

The H.C.M.[1-2] uses stopped delay instead of overall delay to determine the level of service at signalized intersections. The model has a uniform delay term and an overflow delay term called the incremental delay term. Unlike the Australian and Canadian delay models, the H.C.M. assumes a fixed analysis time period of 15 minutes regardless of the actual congestion period. The coefficients of the uniform and incremental terms differ from other time dependent delay models due to the conversion factor and the fixed analysis period of 15 minutes.

Another difference of the 1985 H.C.M. delay model from the Australian and Canadian delay models is the x^2 (n = 2) calibration term. The H.C.M. uses a calibration term to obtain better results in estimating delay for under-saturated conditions. The H.C.M. delay model overestimates delay at high degrees of saturation because of the calibration term in the incremental delay component of the model.

The 1985 H.C.M. delay model, with its uniform and incremental components, is expressed as follows:

$$d_d = 0.38\frac{C(1-\lambda)^2}{(1-\lambda x)} + 173x^2\left[(x-1)+\sqrt{(x-1)^2+\frac{16x}{c}}\right] \qquad (20)$$

where

d_s = average stopped delay.

1994 H.C.M. Delay Model

In 1994, the Transportation Research Board released the latest version of the Highway Capacity Manual.[(2)] which made significant changes to their model. These have been have been discussed by Panesdovus et al. [(27)] and Strong. [(28)] The new delay model is represented as:

$$d_s = 0.38\frac{C(1-\lambda)^2}{\{1-\lambda[Min(x,1.0)]\}} + 173x^2\left[(x-1)+\sqrt{(x-1)+\frac{mx}{c}}\right] \qquad (21)$$

where

m = an incremental calibration term representing the effect of arrival type and. degree of platooning.

Akcelik's Alternative Delay Model

Akcelik [(29)] proposed an alternative to the H.C.M. model. His model gives delay values close to the H.C.M. delay model when the degree of saturation is less than 1.0, and remains asympotic to the deterministic over-saturation line for x values greater than 1.0.

Akcelik's alternative model does not incorporate a x^2 calibration term and the second delay term is set to zero when the degree of saturation is below 0.5. Also the delay parameter *k* is equal to 1.0 in alternative model and 0.5 in the H.C.M. delay model. The alternative model for the 15 minutes fixed analysis period is expressed as:

$$d = 0.5\frac{C(1-\lambda)^2}{(1-\lambda x)} + 225\left[(x-1)+\sqrt{(x-1)^2+\frac{32(x-0.5)}{c}}\right] \quad (22)$$

When applying the conversion factor to the alternative model, Equation (20) becomes:

$$d_s = 0.385\frac{C(1-\lambda)^2}{(1-\lambda x)} + 173\left[(x-1)+\sqrt{(x-1)^2+\frac{32(x-0.5)}{c}}\right] \quad (23)$$

Akcelik's Generalized Delay Model.

Akcelik notes that existing time dependent delay models have more or less the same form. They can be considered as variations of one another, and expressed in a general form. A generalized time dependent delay model is proposed by Akcelik [29-30] as follows:

$$d = 0.5\frac{C(1-\lambda)^2}{(1-\lambda x)} + 900Tx^n\left[(x-1)+\sqrt{(x-1)^2+\frac{m(x-x_0)}{cT}}\right] \quad (24)$$

and

$$x_0 = a + bsg \quad (25)$$

where

m, n, a, b = calibration parameters,

sg = capacity per cycle.

The Australian, Canadian, 1985 H.C.M., 1994 H.C.M, TRANSTY 8 and Akcelik's alternative delay model can be derived from Equations (24) and (25) by setting appropriate values for the calibration parameters n, m, a, and b.

Burrow's Generalized Delay Model

Burrow [31] presented a more general form of the Akcelik's generalized delay model. It covered the work by Kimber and Hollis. His generalized form included an additional term alpha in the over-saturation part of the equation.

$$d = 0.5\frac{C(1-\lambda)^2}{(1-\lambda x)} + 900Tx^n\left[(x-1)+\alpha+\sqrt{(x-1)^2+\frac{m(x+\beta)}{cT}}\right] \quad (26)$$

where

m, n, α,β = calibration terms.

Appropriate values for the calibration terms m, n, α and β give the Australian, Canadian, H.C.M., TRANSTY 8, Alternative to H.C.M. and TRRL delay models. The Australian and TRRL delay models have non-zero values for the term β. The unique property of the TRRL model is that it contains a non-zero value of α. The term α is a function of γ which was used for C in the Kimber and Hollis notation to avoid confusion with C, the cycle time. The parameter γ describes the arrival and departure patterns of traffic on the approach and it is the same as the delay parameter *k* in other time dependent delay models.

Suggested Calibration Term for H.C.M. Delay Model

A calibration term x^2 was introduced to the 1985 H.C.M delay model to reduce predicted delays in under-saturated conditions. To achieve this, a non-linear calibration term was desired and the x^2 parabolic function was selected. Because of the non linearity in x, the basic coordinate transformation equation presented by Kimber and Hollis could not be easily adjusted internally to its model form. [32]

The calibration term x^2 works very well for under-saturated conditions but it overestimates delay at high degrees of saturation. Because of the parabolic property of the x^2 function the H.C.M. model diverges from the deterministic line for x values above 1.0. This divergence is contrary to the basic time dependent delay model, which forces the delay curve to be asymptotic to deterministic line.

This paper proposes non-linear function to replace the calibration term x^2 to better describe delay at high degrees of saturation and to improve the estimation of delay for under-saturated conditions. An appropriate curve for the calibration term is obtained from the combination of exponential and parabolic functions. Theses functions alone cannot perform the aim revealed above. The suggested calibration term x^n for the H.C.M. delay model is given in Equation (27).

$$n = \frac{e^{\frac{1}{7}x} - 1}{x^{1.75}} \tag{27}$$

The comparison of the calibration terms x^2 and x^n indicated that the proposed x^n term works well for both under-saturated and over-saturated conditions.

DEVELOPMENT OF TIME DEPENDENT DELAY MODELS FOR VARIABLE DEMAND AND TIME CONDITIONS

Arrival and service characteristics at a signalized intersection determine the level of delay and queuing on the approach. For the time dependent delay models, the arrival and service characteristics are described by a delay parameter *k*. The analytical equations developed for estimating this delay parameter *k* are obtained by using queuing analysis methods or simulation models. This paper develops two different forms of the delay parameter *k*, which are functions of degree of saturation *x* and analysis time period *T*, using simulation model TRAF-NETSIM.

Akcelik's Delay Parmeter

Akcelik and Rouphail,[33,34] used a cycle by cycle simulation model to develop an expression for the delay parameter *k* as a function of capacity per cycle. Two delay parameters *k* and x_0 were derived for random and platoon arrivals using the steady state delay model given in Equation (28). Then, by using a coordinate transformation technique these two delay parameters were applied to the time dependent delay model presented in Equation (29).

$$d_s = \frac{k(x - x_0)}{Q(1 - x)} \tag{28}$$

where

d_s = stochastic steady state delay in sec,

Q = capacity vph.

$$d_2 = 900T\left[(x-1) + \sqrt{(x-1)^2 + \frac{8k(x - x_0)}{QT}}\right] \tag{29}$$

for $x > x_0$ (zero otherwise)

where

d_2 = overflow delay term of the time dependent delay model.

The expression *k* developed by Akcelik and Rouphail incorporates the ratio of variance-to-mean arrivals per cycle I_u, at the upstream approach. For random arrivals I_u equals 1 and the delay parameter *k* is given by:

$$k = 1.22sg^{-0.22} \tag{30}$$

This expression is applied only when the degree of saturation x is greater than 0.5, and the k values in Equation (30) range from 1 to 0.5 for sg values in the range of 3 to 60 vehicles per cycle.

For platooned arrivals, the delay parameter k is not only a function of capacity per cycle but also a function of the magnitude of the platooning and the cycle to cycle variation in the arriving stream. The magnitude of platooning, PIP, is equivalent to the proportion of vehicles stopped at the upstream intersection and is given by:

$$PIP = \begin{cases} \dfrac{\left(1-\dfrac{g_u}{C}\right)}{\left(1-\dfrac{g_u}{C}x_u\right)} & \text{for } x_u \le 1.0 \\ 1.0 & \text{for } x_u > 1.0 \end{cases} \tag{31}$$

Then, the delay parameter k for platooned arrivals is expressed as follows:

$$k = \begin{cases} (1.22-0.527PIP)\,(sg)^{-0.22} & \text{for } x_0 = 0.5 \\ \dfrac{0.302}{1-PIP}(sg)^{-0.22} & \text{for } x_0 > 0.5 \end{cases} \tag{32}$$

Tarko / Rouphail / Akcelik's Delay Parameter

Tarko et al. [12] developed a model using the difference in between the upstream and downstream intersection capacities to describe the delay parameter k. The random overflow delay approaches zero when the capacity is less or equal to the capacity at the downstream intersection. The model is presented in Equation (33).

$$k = k_0 f \tag{33}$$

where

k_0 = model parameter for an isolated intersection,
f = The adjustment factor for upstream conditions as a function of the difference between the upstream and downstream capacities.

The f term in the model is expressed as follows:

$$\begin{aligned} f &= 1 \quad && when \quad (sg)_u >> (sg)_d \\ 0 < f &< 1 \quad && when \quad (sg)_u > (sg)_d \\ f &= 0 \quad && when \quad (\mathrm{sg})_{\mathrm{u}} < (sg)_d \end{aligned} \tag{34}$$

where
$>>$ = much greater than,
$(sg)_u$ = the upstream capacity in vehicles per cycle,
$(sg)_d$ = the upstream capacity in vehicles per cycle.

After calibration, the final form of the delay parameter *k* is:

$$k = 0.408\left\{1 - e^{-0.5[(sg)_u - (sg)_d]}\right\} \tag{35}$$

The calibrated delay parameter *k* is true for $(sg)_u > (sg)_d$ and $x > (sg)_d / 100$. When these conditions are not met, the delay parameter *k* becomes zero. While Equation (36) represents the stochastic steady state form of a delay model for the developed delay parameter *k*, Equation (37) presents time dependent form of the model.

$$d_s = \frac{0.408\left\{1 - e^{-0.5[(sg)_u - (sg)d]}\right\}\left(x - \frac{sg}{100}\right)}{Q(1-x)} \tag{36}$$

In the Equation (37), m_c is substituted for $(sg)_u$, and defined as maximum number of arrivals per cycle.

$$d_2 = 900T\left[(x-1) + \sqrt{(x-1)^2 + \frac{3.3\left(1 - e^{-0.5[(m_c)-(sg)_d]}\right)\left(x - \frac{sg}{100}\right)}{QT}}\right] \tag{37}$$

Daniel's Delay Parameter

Daniel [35,36] presented a model for the three signal controller types to express the delay parameter *k* at signalized intersections. She calibrated the delay parameter *k* by setting an estimate of the measured incremental delay to the time dependent model and solving for *k*. The delay parameter *k* was expressed as a function of degree of saturation, and an exponential equation was used. Her model for three controller types is presented in Equation (38).

$$k = e^{\beta_0} x^{\beta_1} \tag{38}$$

The results of her study show that the delay parameter *k* for pre-timed control has the highest values, ranging between 0.39 and 0.02, when the degree of saturation varies from 0.5 to 1.0. The delay parameter *k* for semi-actuated and fully-actuated control ranges from 0.197 to 0.005 and from 0.313 to 0.016 respectively.

To minimize the effect of the large *k* values at low and high degrees of saturation, the delay parameter *k* was developed only under traffic conditions for degrees of saturation *x* between 0.5 and 1.0.

The methodology for developing the delay parameters for time dependent delay models in this research, expands upon Daniel's methodology.

Development of the Model for Variable Demands

Various methodologies for dealing with variable demands have been performed by Akcelik [34,37] and Ceder et al.[38-39] The approach taken in this paper was to define variable demand as given variations over time of degree of saturation *x* and to model this with the simulation TRAF-NETSIM.

The intersection simulated consisted of one lane for every approach. To avoid the effect of spillback, the link lengths of the intersection were set to the maximum lengths allowable in the simulation. Queue spillback still occurred at degrees of saturation was between 1.3 and 1.5. The intersection was considered as a micro node to take account of the effects of spillback on total delay. Turning and pedestrian traffic were excluded from the simulation to eliminate mixed effects.

The intersection was operated with a cycle length of 90 seconds and two phases. The yellow and all red intervals were 3 and 2 seconds for all approaches respectively, displayed green times were 50 seconds for the major and 30 seconds for the minor approaches. A start-up lost time of 2 seconds, a mean discharge headway of 2 seconds per vehicle (saturation flow rate of 1800 vphpl) and a free flow speed of 30 mph were used in the simulation runs.

Entry link volumes varied from 60 vph to 900 vph for the minor approaches and from 100 vph to 1500 vph for the major approaches. Thus, the degree of saturation ranged between 0.1 and 1.5 for both the major and minor approaches of the intersection. The percentage of trucks and carpools for all links was given as 5 % and 0% respectively.

In the initial experiment, the duration of the each simulation run was 15 minutes and for each entry link volume, 10 simulation runs with different random seed numbers were made. The random seed numbers were not varied from one degree of saturation to the other, and kept constant during multiple runs to obtain identical traffic movements.

When the degree of saturation x was less than 0.5 and greater than 1.0, the variation in k values was so great that supplementary simulation runs were made to obtain a significant amount of data. Thus, a total of 182 simulation runs were made to develop a delay model that is suitable for variable demand conditions.

Development of Delay Parameter *k*

Time dependent delay models consist of the two delay components for uniform and overflow delay. The uniform delay is the first term of Webster Equation (2) for under-saturated conditions and from Equation (15) for over-saturated conditions. These equations perfectly estimate uniform delay, so there was no further examination of the uniform delay term.

The overflow delay term represents the additional delay that results from temporary and persistent over-saturation conditions. It becomes a combination of the random overflow and the continuous overflow delay terms. While random overflow delay may occur at all degrees of saturation, continuous overflow delay solely occurs when the degree of saturation is greater than 1. For this reason, the random overflow delay is a core term to model the delay parameter *k*. Figure 1 graphically illustrates these delay terms.

$$d = d_u + d_o \tag{39}$$

Where

d = average overall delay,
d_u = uniform delay,
d_o = overflow delay.

$$d_o = d_{ro} + d_{co} \tag{40}$$

where

d_{ro} = random overflow delay,
d_{co} = continuous overflow delay.

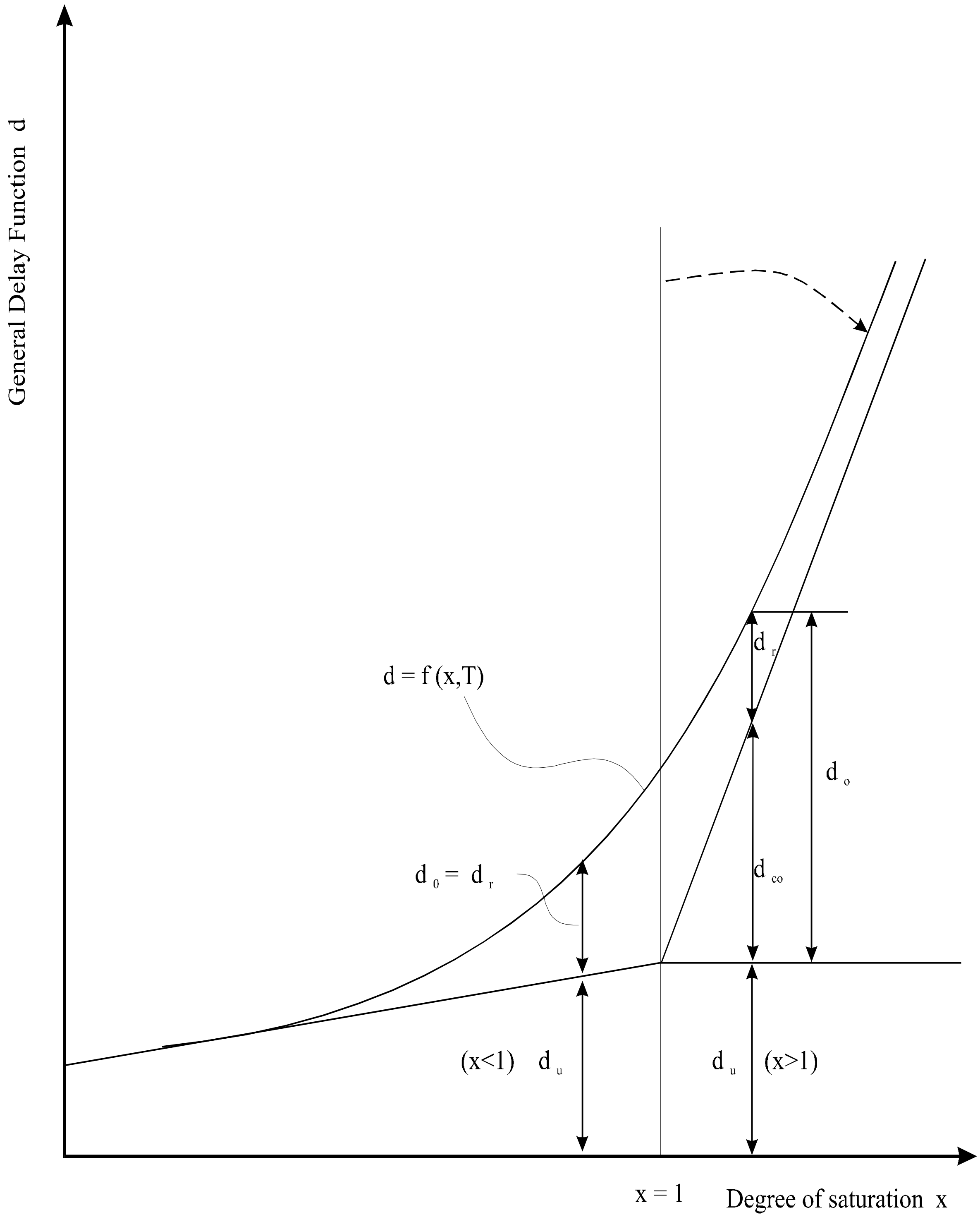

Figure 1 A General Delay Function for Delay Components [19]

For under-saturated conditions continuous overflow delay is zero and the simulated delay equals the sum of the uniform and random overflow delays. Estimate of random overflow delay holds true only if the simulated delay is greater than the estimate of uniform delay. When the simulated delay is less than uniform delay, that random overflow delay is zero.

For over-saturated conditions, random overflow delay is estimated as the difference between the simulated delay and the sum of uniform delay and continuous overflow delay. The value of the delay parameter *k* for both conditions is obtained by substituting the simulated random overflow delay in the time dependent equation.

For this paper, the Canadian delay model was selected to solve for *k* for given traffic conditions. 728 data points were obtained from the 182 simulation runs, but 94 of these were excluded because the simulated delay was either less than the uniform delay or the sum of the uniform delay and the continuous overflow delay.

From the simulations, the variation among the calculated *k* values for low and high degrees of saturation was large because the random overflow delay is not the dominant component at these degrees of saturation. The variation for low degrees of saturation is greater than that for high degrees of saturation even though the random delay associated with these degrees of saturation is small. Therefore, in the modeling of the delay parameter *k*, an upper and lower boundary value was needed to minimize the effect of large *k* variables. The lower and upper boundary values of delay parameter *k* were selected as 0 and 1.5.

In the literature, specific information for maximum and minimum boundary values of *k* was not available. The Australian delay model uses a *k* of 1.5, a high value that is compensated for by the x_0. Kimber and Daly [(40)] observed queue lengths at different sites to calculate *k* values. They found a maximum *k* of 1.5 at one of their sites. A choice of a *k* of 1.5 as an upper boundary value, therefore, is a realistic assumption. Figure 19 shows the fitted k curve after the reduction of data for the upper boundary value of 1.5.

The *k* values obtained according to the simulation results gave the best fit with the following second-degree parabolic function.

$$k = 0.8095x^2 - 1.4141x + 1.1335 \tag{41}$$

Equation (41) is simplified to:

$$k = 0.8x^2 - 1.4x + 1.1 \tag{42}$$

The k values in Equation (42) have extreme values with a value of 0.488 at x of 0.9 and a value of 0.968 at x of 0.1 for degrees of saturation between 0.1 and 1.5. The model for the delay parameter k is applicable to all degrees of saturation and for variable demand conditions.

Development of the Model for Variable Time Conditions

One of the delay parameters which determines the level of random overflow delay is k variable. It should vary with demand, capacity and analysis time period. The variation in the traffic flow describes either demand or capacity as well as degree of saturation. Using a fixed k value may result in overestimation or underestimation of overflow delay for variable demands.

The expression of k given in Equation (30) and developed by Akcelik and Rouphail determines the level of overflow delay by considering the effect of capacity per cycle. This expression seems to work well when degrees of saturation are greater than 0.5 and the sg values are in the range of 3 to 60 vehicles per cycle. Therefore, an expression of k as a function of capacity was not developed. On the other hand, the analysis time period is an essential variable for determining the level of overflow delay at signalized intersections, but their seem to be no studies associating the delay parameter k and with the analysis time period T.

A new form of the delay parameter k that is a function of analysis time period was developed. The simulated intersection to develop this for k had same configurations with the intersection used in the previous study. The analysis time period varied from 15 minutes to 1 hour and 15 runs were made for each analysis time period. In each simulation run, a different random seed number was used to eliminate similar driver and vehicle characteristics. They were kept constant between runs, however, to have identical traffic movements when different analysis time periods were being compared.

Four analysis time periods of 0.25 h, 0.50 h, 0.75 h and 1.0 h were considered. A total of 60 simulation runs were performed and a total of 240 data points were acquired.

Development of Delay Parameter *k*

By following the same methodology explained above, k variables were calculated for each analysis time period. After the all simulation runs, all k variables computed were positive and less than 1.5. Thus, all data was available for the modeling of k.

The mean values of the calculated k variables were for the analysis periods are explained by a logarithmic and power curve. The developed models for the analysis time periods are statistically investigated below. The models are expressed in Equations (43) and (44).

$$k = 0.0545 Ln(T) + 0.6915 \tag{43}$$

$$k = 0.6923 T^{0.0844} \tag{44}$$

The delay parameter k in Equation (43) varied from 0.5282 for an analysis time period of 0.05 hours to 0.6915 for an analysis time period of 1 hour. The other values of k in Equation (44) ranged from 0.5376 to 0.6923 for the same analysis time periods. Table 1 and Figure 3 show the relationship between analysis time periods and k values for both equations. Note that Equation (43) and Equation (44) refer to k_1 and k_2 respectively and these provide Model 1 and Model 2.

COMPARISON OF MODELS

The delay models, Model 1 and Model 2, developed in this paper were compared to the H.C.M., the Australian, the Canadian, and the deterministic delay models. The degree of saturation ranged between 0.1 and 2.0. The analysis time periods ranging from 15 minutes to 1 hour. In comparing the delay models, only the overflow delays were considered since all of the delay models have the same expression for the uniform delay.

The overflow delays for an analysis period of 15 minutes are given in Table 2 and shown in Figure 4. The highest delay estimate was given by Model 2 for the under-saturated traffic condition. The Australian delay model gave lower values and predicted zero overflow delay at x values below 0.7, because of its x_0 parameter. When x was at 1.0, all the models were similar with estimated overflow delays around 40 seconds.

For the over-saturated condition, Model 2 and the Canadian delay model gave similar overflow delays. The estimated values given by Model 1 and the Australian model for the overflow delay were almost the same, except that the overflow estimation of Model 1 was a little bit lower than that of the Australian model for the degree of saturation x between 1.1 and 1.5. When the degree of saturation was between 1.6 and 2.0, however, the Australian delay model gave higher values than Model 1.

For the values of the degree of saturation between 1.0 and 2.0, the H.C.M delay model predicted much higher delays than any of the others and diverged from the deterministic line. Yet, to be asymptotic to the deterministic over-saturated line is an important characteristic of time dependent delay models. The Australian, Canadian and the newly developed delay models predicted overflow delays of around 460 seconds at the degree of saturation 2.0 whereas the H.C.M. delay model estimated an overflow delay at 1828 seconds.

Table 1. Analysis Time Periods and Developed Delay Parameters

Analysis Time Periods (T)	K_1 Values	k_2 Values
0.05	0.5282	0.5376
0.10	0.5660	0.5700
0.15	0.5881	0.5899
0.20	0.6038	0.6044
0.25	0.6159	0.6159
0.30	0.6259	0.6254
0.35	0.6343	0.6336
0.40	0.6416	0.6408
0.45	0.6480	0.6472
0.50	0.6537	0.6530
0.55	0.6589	0.6582
0.60	0.6637	0.6631
0.65	0.6680	0.6676
0.70	0.6721	0.6718
0.75	0.6758	0.6757
0.80	0.6793	0.6794
0.85	0.6826	0.6829
0.90	0.6858	0.6862
0.95	0.6887	0.6893
1.00	0.6915	0.6923

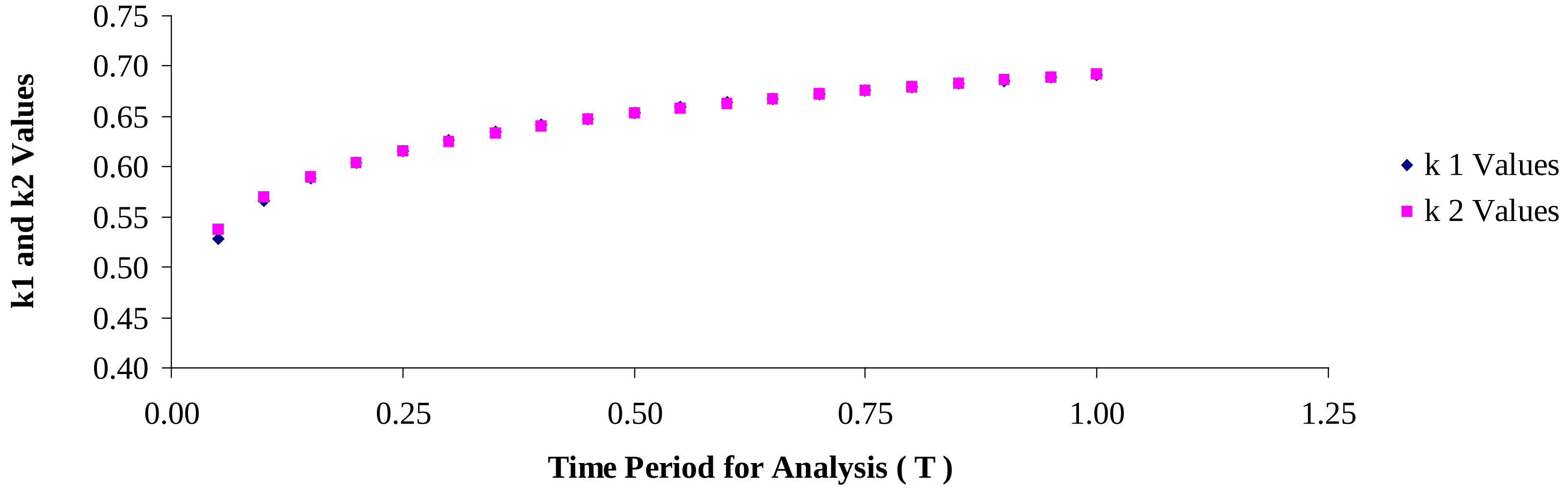

Figure 3 k_1 and k_2 Values for Analysis Time Periods (T)

Table 2. Predicted Overflow Delays (Analysis Time Period T = 0.25 h)

Degree of Saturation (x)	Analysis Time Period (T)	Model 1	Model 2	Canadian	Australian	H.C.M.	Deterministic
0.1	0.25	0.77	0.49	0.40	0.00	0.00	0.00
0.2	0.25	1.53	1.11	0.90	0.00	0.04	0.00
0.3	0.25	2.30	1.89	1.54	0.00	0.14	0.00
0.4	0.25	3.17	2.92	2.38	0.00	0.38	0.00
0.5	0.25	4.24	4.35	3.54	0.00	0.89	0.00
0.6	0.25	5.74	6.42	5.25	0.00	1.89	0.00
0.7	0.25	8.11	9.66	7.93	0.32	3.89	0.00
0.8	0.25	12.45	15.18	12.63	5.54	8.08	0.00
0.9	0.25	21.42	25.48	21.82	16.51	17.67	0.00
1.0	0.25	40.25	44.67	40.25	38.75	40.25	0.00
1.1	0.25	71.37	74.47	70.34	72.44	85.11	45.00
1.2	0.25	110.18	111.48	108.00	112.07	155.52	90.00
1.3	0.25	152.46	152.06	149.12	154.19	252.02	135.00
1.4	0.25	196.36	194.37	191.82	197.45	375.97	180.00
1.5	0.25	241.12	237.60	235.33	241.29	529.48	225.00
1.6	0.25	286.43	281.35	279.28	285.48	714.96	270.00
1.7	0.25	332.12	325.42	323.51	329.87	934.95	315.00
1.8	0.25	378.08	369.71	367.93	374.40	1192.08	360.00
1.9	0.25	424.27	414.15	412.46	419.02	1488.99	405.00
2.0	0.25	470.65	458.70	457.09	463.72	1828.35	450.00

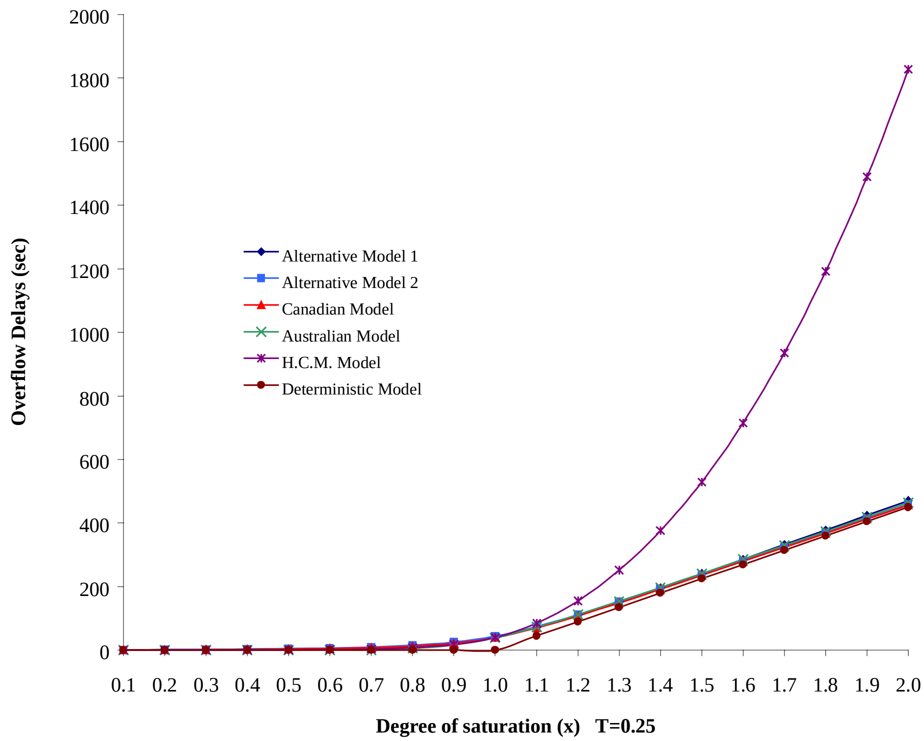

Figure 4. Comparison of Overflow delays for an Analysis Time of 0.25 h.

With the comparison of analysis time periods of 0.50h, 0.75h and 1.0h, the H.C.M. delay model was excluded because it used a fixed analysis time period of 0.25h. The results for the remaining models were similar. The relative values of the overflow delay for each model and analysis time period did not change significantly when the degree of saturation x was between 0.1 and 0.5. For all analysis time periods all the models had responses similar their response for the analysis period of 0.25h. Examples of the comparisons for the analysis period of 1.00 h. are given in Table 3 and Figure 5.

DELAY MODELS FOR VARIABLE DEMAND, TIME AND OVERSATURATED CONDITIONS

In the previous section the new delay models were compared with existing models widely used for delay estimation at signalized intersections. The developed models showed excellent results for a given traffic conditions, but the comparisons did not represent the performance of the models when the demand flow profile changes. The TRAF-NETSIM simulation program was used to verify the new models, for variable demand, time and over-saturated conditions. The delays simulated by TRAF-NETSIM and the delays estimated by the analytical models were statistically compared using linear regression analysis. The results showed that the delays estimated by the analytical models were in close agreement with those simulated by TRAF-NETSIM.

Table 3 Predicted Overflow Delays for an Analysis Time of 1.0 h.

Degree of Saturation (x)	Analysis Time Period (T)	Model 1	Model 2	Canadian	Australian	Deterministic
0.1	1.00	0.77	0.55	0.40	0.00	0.00
0.2	1.00	1.53	1.25	0.90	0.00	0.00
0.3	1.00	2.32	2.13	1.54	0.00	0.00
0.4	1.00	3.20	3.31	2.39	0.00	0.00
0.5	1.00	4.30	4.96	3.59	0.00	0.00
0.6	1.00	5.87	7.40	5.36	0.00	0.00
0.7	1.00	8.47	11.39	8.27	0.32	0.00
0.8	1.00	13.65	18.94	13.87	5.79	0.00
0.9	1.00	27.44	37.18	28.03	20.29	0.00
1.0	1.00	80.50	94.72	80.50	77.50	0.00
1.1	1.00	215.01	224.05	213.40	216.69	180.00
1.2	1.00	383.21	387.77	380.44	385.66	360.00
1.3	1.00	559.05	560.80	555.17	561.10	540.00
1.4	1.00	737.42	737.04	732.39	738.66	720.00
1.5	1.00	916.96	914.71	910.67	917.15	900.00
1.6	1.00	1097.16	1093.13	1089.52	1096.12	1080.00
1.7	1.00	1277.79	1271.99	1268.68	1275.38	1260.00
1.8	1.00	1458.74	1451.13	1448.05	1454.82	1440.00
1.9	1.00	1639.94	1630.46	1627.56	1634.38	1620.00
2.0	1.00	1821.35	1809.91	1807.17	1814.03	1800.00

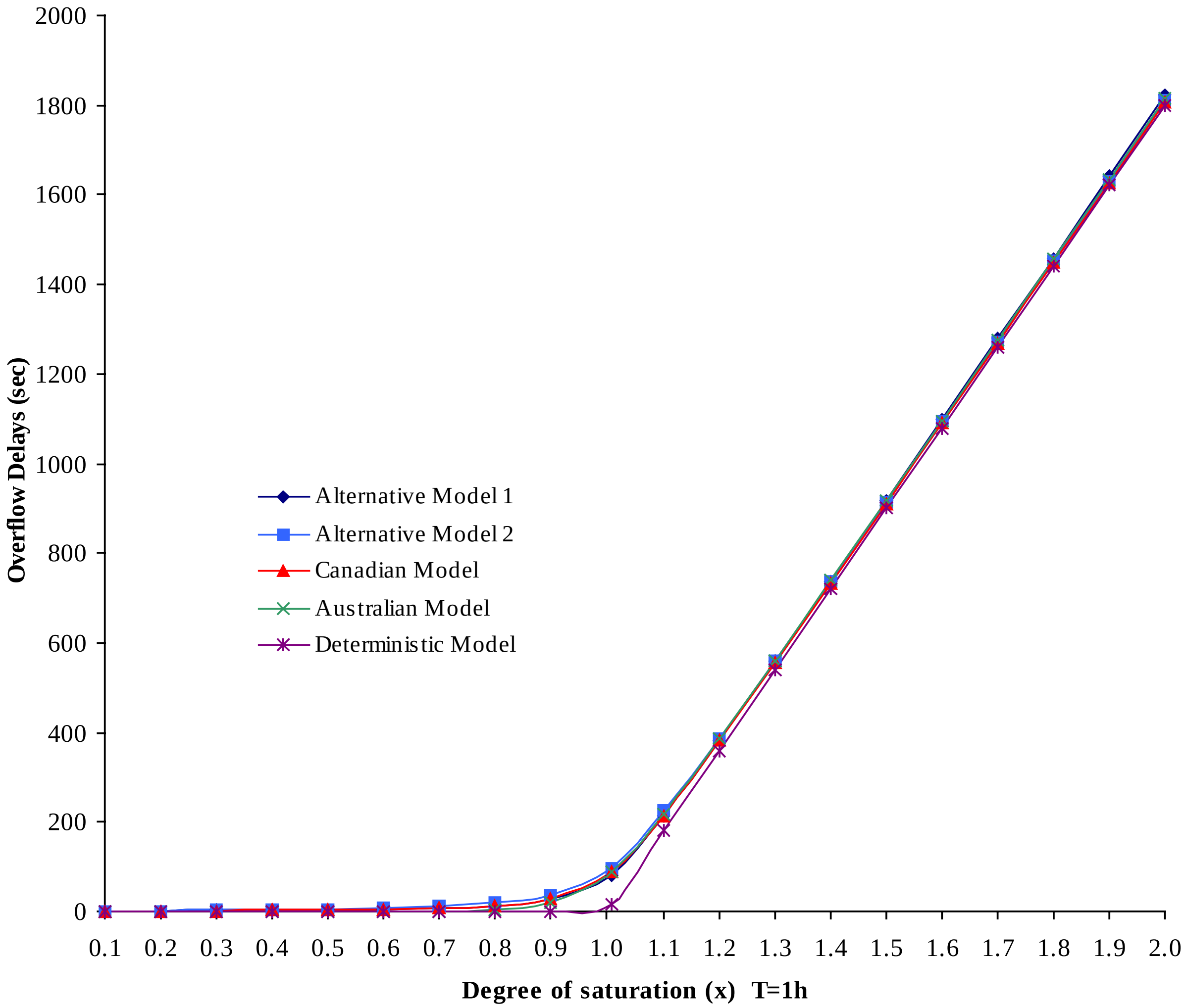

Figure 5. Comparison of Overflow delays for an Analysis Time of 1.00 h.

Scenarios Analyzed

In the validation of the models, the period of simulation of 60 minutes was divided into three consecutive time periods to reflect the demand variation in the traffic flow. The first period was an initial time period and had duration of 5 minutes with a constant degree of saturation 0.7 for all cases. During this period, the intersection was initialized without transferring a queue to the second period. The second period, which was the actual analysis period, had one of four over-saturated traffic conditions with the degree of saturation ranging between 1.1 and 1.4, and six time periods of analysis varying from 5 minutes to 30 minutes. The third and last period was the dissipation period, with degrees of saturation of 0.5 and 0.7. The duration of this period depended on the duration of the second period, as it was necessary to dissipate the queues that had built up over the second period.

Table 4 summarizes the all simulation scenarios discussed above. Each of these scenarios was replicated by changing the random seed numbers. Different random seed numbers yielded different event sequences, and different delay estimates.

Table 4. Simulation Scenarios

Time Periods	Degree of Saturation	Simulation Time	Number of Setups
Initialization Period	0.7	5	1.1 = 1
Analysis Period	1.1, 1.2, 1.3, and 1.4	5, 10, 15, 20, 25, 30	4.6 = 24
Dissipation Period	0.7 and 0.5	-------------	2.1 = 2

Originally, this study planned to analyze scenarios that included a degree of saturation of 1.5, but initial results showed that queue spillback in the simulation was affecting the outputs. This occurred for a degree of saturation of 1.4 and analysis time periods of 25-30 minutes and for a degree of saturation of 1.5 and analysis time period between 15 minutes and 30 minutes. These scenarios, therefore, were excluded.

Comparison of Results

In the simulation runs, average maximum delays were considered because TRAF-NETSIM uses the path trace method for measuring individual delays from arrival to departure time, even if the latter occurred after the end of the analysis time period. In over-saturated conditions, additional delays occur after the end of the analysis period for vehicles that had arrived during the analysis period and are still in the queue at the end of the analysis period. In practice, this additional delay is ignored and only the delay occurring during the analysis period is considered. The concept of the average maximum delay is given in detail by Akcelik and Rouphail [(41)].

The estimated delays by the models were compared with the simulated delays. The results are shown in Table 5 for the traffic condition 0.7-O/S-0.5, and in Table 6 for the traffic condition 0.7-O/S-0.7. It was noted that changes in the degree of saturation during the the dissipation period had no significant effect on average maximum delays as long as these conditions were under-saturated.

Statistical Analysis

The data obtained from simulation results for traffic conditions 0.7-O/S-0.5 and 0.7-O/S-0.7 were grouped for a linear regression analysis. Scatterplots of predicted delay versus the means of TRAF-NETSIM delay replications indicated a close linear relationship with most of the data.

In the linear regression analyses, the statistical comparison was conducted with the models as the dependent variable and the simulated output as the independent variable. Each of the delay models developed was analyzed with and without an intercept. In the no intercept case the regression equation was forced through zero because the delay by the model should be zero when simulation generates zero delay. The other case was considered in since it presents the actual magnitude of the differences between them without effecting originality of the data.

The regression analysis results between the delay models and TRAF-NETSIM for the intercept and no intercept cases indicated that the delays simulated by TRAF-NETSIM are explained by the models' estimates with a correlation of 99 %.

The regression analysis results were very encouraging because the constant coefficient is not substantially different from the null, with the regression results for the intercept and no intercept cases were almost the same for both delay models. For the cases with and without an intercept of Model 1, the linear relations are given in Equations (45) and (46). For Model 2, the linear relations are given by Equations (47) and (48).

$$Y = -0.05 + 0.9850X \tag{45}$$

$$Y = 0.00 + 0.9847X \tag{46}$$

$$Y = 1.18 + 0.9801X \tag{47}$$

$$Y = 0.00 + 0.9862X \tag{48}$$

where

Y = Total delay from models developed

X = Total delay from TRAF-NETSIM.

The regression analysis results showed that the estimates of delay models presented in this paper provide an excellent statistical fit to simulated delays with an r-square and correlation value of 0.99 at a confidence level of 95 percent.

Table 5. Comparisons of Results for Condition 0.7 – O/S – 0.5

Analysis Time Period (minutes)	Degree of Saturation (x)	Model1 Estimates	Model2 Estimates	Netsim Estimates
5	1.1	46.66	47.16	45.71
10	1.1	64.26	65.59	60.68
15	1.1	80.54	82.55	77.54
20	1.1	96.35	98.92	94.34
25	1.1	112.10	115.15	111.21
30	1.1	127.50	130.97	128.05
5	1.2	59.28	59.13	56.48
10	1.2	90.64	91.00	86.29
15	1.2	121.01	121.77	117.25
20	1.2	151.17	152.24	148.58
25	1.2	181.60	182.92	179.87
30	1.2	211.61	213.14	211.44
5	1.3	73.14	72.32	73.39
10	1.3	119.15	118.66	120.08
15	1.3	164.23	164.01	164.63
20	1.3	209.19	209.18	210.15
25	1.3	254.64	254.80	254.08
30	1.3	299.52	299.83	286.04
5	1.4	87.64	86.15	88.95
10	1.4	148.58	147.32	157.99
15	1.4	208.52	207.45	218.12
20	1.4	268.37	267.46	271.18
25	1.4	328.91	328.13	307.13
30	1.4	388.71	388.05	330.80

Table 6. Comparison of Results for the Condition 0.7 – O/S – 0.7

Analysis Time Period (minutes)	Degree of Saturation (x)	Model 1 Estimates	Model 2 Estimates	Netsim Estimates
5	1.1	46.66	47.16	46.68
10	1.1	64.26	65.59	62.14
15	1.1	80.54	82.55	79.72
20	1.1	96.35	98.92	96.83
25	1.1	112.10	115.15	114.24
30	1.1	127.50	130.97	131.53
5	1.2	59.28	59.13	58.9
10	1.2	90.64	91.00	90.19
15	1.2	121.01	121.77	122.61
20	1.2	151.17	152.24	155.17
25	1.2	181.60	182.92	187.95
30	1.2	211.61	213.14	221.15
5	1.3	73.14	72.32	77.42
10	1.3	119.15	118.66	126.23
15	1.3	164.23	164.01	173.57
20	1.3	209.19	209.18	222.81
25	1.3	254.64	254.80	268.93
30	1.3	299.52	299.83	300.98
5	1.4	87.64	86.15	97.49
10	1.4	148.58	147.32	167.39
15	1.4	208.52	207.45	232.26
20	1.4	268.37	267.46	290.29
25	1.4	328.91	328.13	324.89
30	1.4	388.71	388.05	348.74

CONCLUSIONS

Estimation of delay at signalized intersections is a complex process and depends on various variables. Of all the variables impacting delay, degree of saturation (x) and analysis time period (T) are two of the most important. Although the influence of these two variables on delay estimations is known and has been widely discussed, there has been less effort to adequately represent them in delay models. The general methodology of this research has been to develop improved delay models that better represent the effects of variable demand and analysis time period.

Two analytical delay models for signalized intersections that consider the variation in traffic flow and time period have been developed. Unlike existing delay models, the delay parameter k is expressed as a function of degree of saturation (x) and analysis time period (T) in both models. A comparative study of the new models against the existing models verified the new models.

The TRAF-NETSIM microscopic simulation model was used for over-saturated conditions to validate the delay models. The results obtained from the simulation and the developed models were statistically analyzed and were in close agreement.

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AUTHORS

Ali Payidar Akgungor is a Research Assistant at Kirikkale University, The Engineering Faculty, Civil Engineering Department, in Yahsihan, Kirikkale/ Turkey

A. Graham R. Bullen is Associate Professor, University of Pittsburgh, School of Engineering, Department of Civil and Environmental Engineering, 949 Benedum Hall, Pittsburgh, PA 15261